Acceleration Transformation in SR

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*thanks to Richard A. Mould

Given:

- An inertial reference frame S' for which events are specified with coordinates (ct', x').
- A uniformly accelerating system of coordinates S (sometimes called an accelerating reference frame) for which events are specified with coordinates (ct, x).

If S and S' coincide at t = t' = 0, then the relationship (transformation) between events in S and S' is given by;

$$x'(x,\tau) = \frac{-1}{\kappa} + \left(x + \frac{1}{\kappa}\right) cosh(\kappa\tau)$$

$$ct'(x,\tau) = \left(x + \frac{1}{\kappa}\right) sinh(\kappa\tau)$$

Where:

- $\kappa = \frac{a}{c^2}$
- $\tau = ct$
- a = uniform acceleration as experienced by an observer at the origin of S (proper acceleration).
- c = speed of light.
- (ct, x) = space time coordinates of the event with respect to the accelerating observer.
- (ct', x') = space time coordinates of the event with respect to the inertial observer.

Substituting gives

$$x'(x,t) = \frac{-c^2}{a} + \left(x + \frac{c^2}{a}\right) cosh\left(\frac{at}{c}\right)$$

$$ct'(x,t) = \left(x + \frac{c^2}{a}\right) sinh\left(\frac{at}{c}\right)$$

If we are only interested in the movement of the origin of the accelerating system in space-time (x always zero) that origin has coordinates with respect to the inertial frame of;

$$x' = \frac{c^2}{a}\left(cosh\left(\frac{at}{c}\right) - 1\right)$$

$$t' = \left(\frac{c}{a}\right) sinh\left(\frac{at}{c}\right)$$

Reference:
Basic Relativity (Chapter 8)
By Richard A. Mould
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If I do some algebra correctly, then, with respect to an observer at the origin of the accelerating system, the space-time coordinates of the origin of the inertial reference frame (x' always zero) is given by;

$$x = \frac{c^2}{a}\left(\frac{1}{cosh\left(\frac{at}{c}\right)} - 1\right)$$

$$t = \left(\frac{c}{a}\right) arcsinh\left(\frac{at'}{c}\right)$$